



## Background on ME 7442: Vibration of Continuous Systems

- Ultimate graduate-level vibrations (and structural dynamics) course offered in ME
- Focused on advanced modeling and analysis of vibration of strings, shafts, beams, membranes, and plates using exact and approximate mathematical methods

ME 7442 Vibration of Continuous Systems  
Fall 2012, MWF 10:05-10:55, Instr. Center 209

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Office hours: Monday & Wednesday 4:30-5:30pm or by email appointment

Pre-requisite: ME 6442 or equivalent, or with the consent of the instructor

Webpages: t-square.gatech.edu

References (no required textbook):

- Meirovitch, L., 2001, *Fundamentals of Vibrations*, McGraw-Hill.
- Meirovitch, L., 1997, *Principles and Techniques of Vibrations*, Prentice Hall.
- Shames, I. H. and Dym, C. L., 1985, *Energy and Finite Element Methods in Structural Mechanics*, Hemisphere Publishing.
- Dym, C. L. and Shames, I. H., 1973, *Solid Mechanics: A Variational Approach*, McGraw-Hill.

## TOPICS

### Review of the Elements of Analytical Dynamics

- Generalized coordinates; the principle of virtual work; d'Alembert's principle; Hamilton's principle; Lagrange's equations
- A brief review example: Vibration of a 3-DOF discrete system

### Transverse Vibration of Strings

- Boundary value problem formulation (Newtonian and Lagrangian)
- Exact solution (free vibration and differential eigenvalue problem, forced vibration)
- Approximate methods (Rayleigh's quotient, Rayleigh-Ritz method, assumed-modes method, Galerkin's method)
- Example problems (with different boundary conditions)

### Longitudinal Vibration of Bars

Fig 1. An extracted portion from the course syllabus improved in Fall 2012.

## Main issue

- Highly mathematical content with mostly symbolic derivations
- Nontrivial for students to connect to physical systems and comprehend the importance and implications of mathematical details

## Class of 1969 Fellows Project

- **Objective:** Enhanced learning and visualization of the importance and physical implications of detailed mathematical derivations covered throughout the course and application of the exact and approximate vibration analysis methods for a widely used physical structure
- **Methodology:** An experimental-theoretical take-home final exam involving hands-on lab session with peers (for engagement and visualization) and independent analysis of collected data along with modeling in the final exam report preparation (for evaluation)

### TAKE-HOME FINAL EXAM

- The final exam will be a take-home exam with an experimental vibration testing component.
- You will participate in the experiment with your peers and take frequency response data of a continuous system (a cantilever under  $n=7$  transverse vibrations) in the lab to become familiar with modal testing procedures and equipment. You will not be graded for your experimental skills.
- All students will eventually be provided the same experimental frequency response functions (MATLAB data) for the take-home exam. Contact me if you have no background to plot the magnitude of an array of complex numbers in MATLAB so I can help you.
- You will be given two weeks to analyze (by yourself) the experimental continuous system using the methods learned in this course. The goal is to predict the natural frequencies, mode shapes, and frequency response functions of the cantilever for multiple vibration modes by using exact and approximate methods. You may use the computer software you prefer (MATLAB, Mathematica, etc.) to simulate your theoretical derivations.
- You will be expected to prepare a neat solution report (your own work) for the continuous system you will have analyzed theoretically and experimentally by addressing the final-exam questions.
- There will be no separate in-class final exam.

Fig 2. Description of the experimental-theoretical take-home final exam in the Fall 2012 syllabus.

## A typical engineering structure as the experimental model

- Cantilevered structures (i.e. structures with fixed-free physical boundary conditions) have been employed in many engineering disciplines for numerous applications for centuries.
- Therefore, a cantilever beam configuration with different lumped mass attachments was chosen as the main experimental test structure.

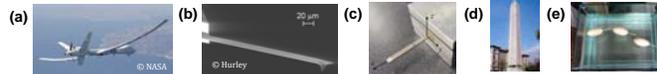


Fig 3. Various cantilevered structures used in (a) aerospace, (b) materials science, (c) mechanical, (d) civil, and (e) ocean engineering systems.

- Frequency response measurements were taken at 7 points on the cantilever to extract the mode shapes and several mobility functions covering the first four vibration modes.
- Additionally, different aspect ratio beams were tested to demonstrate the shear deformation and rotary inertia effects in free boundary conditions (on very soft foundation).

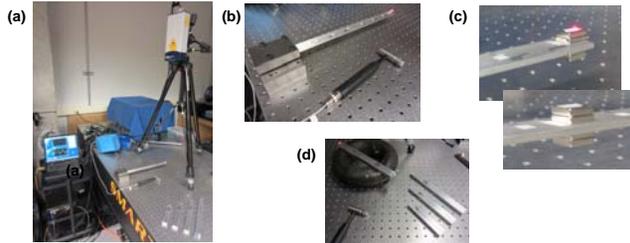


Fig 4. (a) Experimental setup used for modal testing of (b) a cantilever structure with (c) different lumped attachments and (d) aspect ratios to create various final exam problems for exact and approximate modeling.

## Preparation of students for the project

- Students were given special topic lectures on Experimental Modal Analysis (EMA) during the semester.
- Simplified theoretical and numerical versions of the final exam were assigned as homework problems with a focus on strings, bars, and shafts.

### Experimental Modal Analysis (special topic)

- Frequency response functions
- Modal testing methods and equipment
- Modal data extraction and parameter identification
- Frequency response measurement using planar piezoelectric actuators and sensors

Fig 5. Components of special topic lectures on EMA in the syllabus.

## Hands-on experience and collective learning

- The total class (of 12 graduate students) was divided into two groups for two lab sessions (2-3 hours per session).
- Each student was responsible for collecting the frequency response function data between each measurement point and excitation point.
- As stated in the syllabus, students were not graded based on their experimental skills. Therefore, eventually all students were provided the same data set through T-Square for the take-home exam.

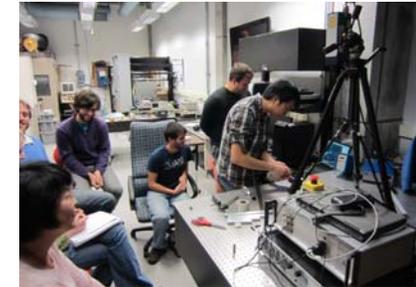


Fig 6. Group of students in Fall 2012 class performing modal testing of a cantilever using an instrumented impact hammer and a laser vibrometer (and sharing the challenge of creating an acceptable impulse excitation by hand).

## Final exam report

- Students were given two weeks to prepare the final exam report.
- They were asked to model the dynamics of the tested configurations using the mathematical tools developed in the course and compare the model results with experimental measurements.

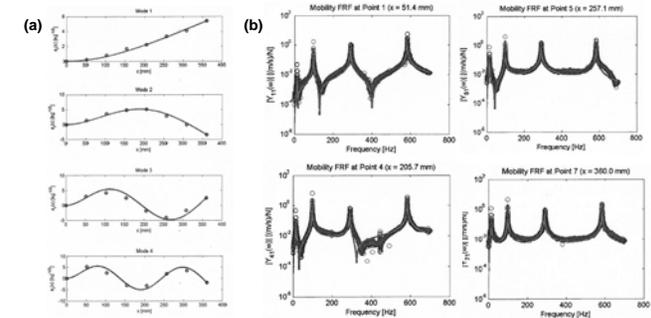


Fig 7. (a) Vibration mode shapes and (b) frequency response functions of the cantilever from a final report (circles: experiment; solid lines: model).

## Outcomes and feedback

- Use of the mathematical methods developed in the course to model a physical system along with hands-on testing experience
- Visualization and enhanced understanding of the importance and implications of exhaustive mathematical derivations
- Engaging team work experience during the experiments and collective learning by discussion
- Appreciation of the experimental realm and physical domain
- Uniformly positive feedback in the CIOS report

## Sustainability

- The project and equipment are sustainable.

## Acknowledgements

- CETL support for the experimental setup (aluminum beams, fixtures, etc.) through the Class of 1969 Teaching Fellows program
- Graduate students in Dr. Erturk's Smart Structures and Dynamical Systems Lab for preparation of the setup