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Multistable vibration energy harvesters: Principle, progress, and perspectives

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ABSTRACT

Vibration energy harvesting is a process by which ambient mechanical energy from environment or host structures is converted into usable energy (usually, but not always, electrical energy). This technology is considered to be a relatively new method for supplying sustainable energy to low-powered sensor networks and electronic devices. Various vibration energy harvesters utilizing piezoelectric, electromagnetic, electrostatic, and triboelectric energy conversion mechanisms were designed and tested to achieve this goal. Meanwhile, one key challenge of such approaches results from their response to the input excitation characteristics, especially in terms of frequency variation. To address that challenge, multistable characteristics commonly exist in mathematical models and physical devices, which can be used for designing vibration isolators, compliant mechanisms, morphing structures, circuits, filters, etc. Currently, multistable vibration energy harvesters have received increasing attention because of their rich nonlinear dynamic characteristics which show benefit for improving efficient vibration energy harvesting bandwidth, i.e. frequency-wise robustness. This paper aims to provide a comprehensive review of the state-of-the-art progress of multistable vibration energy harvesters.

1. Introduction

As a regenerative energy production method, vibration energy harvesting can be categorized as a micro energy generation technique, which converts vibrations induced by human motion, fluid flow, mechanical equipment, among others, into usable electric energy [1–3]. The primary goal is to replace or charge primary batteries for supplying wireless sensors or low-powered embedded devices, which is expected to promote the development of intelligent health monitoring and reduce the chemical waste of conventional batteries [4–6]. While batteries are still used in the large majority of devices, ensuring long-term operation is made very complex due to the self-discharge of such an energy storage system. The multifunctional nature of problem also permits the use of energy conversion device for vibration suppression from the view of energy conservation [7,8]. In addition, the output voltage can be considered as a signal that carries information in some situations, turning the harvester into a special all-in-one self-powered sensor [9,10]. Nevertheless, energy harvesting performance of the device is typically the priority in most applications.

Piezoelectric, electromagnetic, electrostatic, and triboelectric techniques are the most widely selected solutions for the design of a vibration energy converter. In particular, piezoelectric vibration energy harvesters have been widely investigated [11]. In principle, a

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piece of piezoelectric material has two widely used operation modes, namely 31 mode (the polarization and the force are operating in two orthogonal directions) and 33 mode (the polarization and the force are working in the same direction) as illustration shown in Fig. 1(a). When the piezoelectric material is forced to produce a deformation, an equal amount but opposite charges will gather on the two surfaces, which can be used as an alternating source. How to efficiently transfer ambient vibrations to the piezoelectric material as alternating forces over a range of frequencies is the key to vibration energy harvesting.

Via the resonance mechanism, a cantilever-type harvester with piezoelectric materials was proposed as a classic design, which is a composite structure where the piezoelectric materials are placed large mechanical stress, as shown in Fig. 1(b) [12]. Such an approach allows frequency contents that match those usually found in application environment (1–1 kHz) while allowing significant stress or velocity (and thus generated charges) applied to the transducer. The exact governing equations of a cantilever-type piezoelectric energy harvester were derived and experimentally validated by Erturk and Inman [2,12]. Various linear energy harvesters were presented and tested [13,14], and the design for MEMS scale linear harvesters was explored [15]. However, it was found that the linear vibration energy harvester is not efficient if the ambient vibration frequency does not match the harvester's resonant frequency [16, 17]. This is a critical issue when considering that ambient vibration excitation frequency usually features time-varying or broadband characteristics, which make such harvesters to be inefficient in many situations [18–20]. Meanwhile, the inevitable manufacturing tolerances, temperature change (shifting their stiffness and thus resonant frequency) seriously impede actual applications of energy harvesters.

To solve this problem, nonlinearities (intrinsic or induced geometric nonlinearities such as buckling, nonlinear magnetic interactions, impacts, etc.) were brought to the design of harvesters, leading to the concept of nonlinear vibration energy harvesters (NEH) whose stiffness is usually nonlinear [21,22]. The working frequency range of NEH is related to excitation conditions, and is usually in direct proportion to the excitation level. In other words, NEH has more obvious advantage than the linear counterpart when subjected to high-level excitations [23]. The reason is that the force generated by the high-order nonlinear term in the governing model (corresponding to physical property) will work as a larger force on the harvester with a larger displacement or velocity response to higher level excitations.

For the NEH which consists of discrete mass blocks, springs and dampers, the lumped parameter model is usually adopted [24,25]. For the beam or plate type NEH, the distributed parameter model can be firstly derived and then combined with nonlinear terms [26, 27]. In current literature, it was found that the influence of nonlinear terms in the first vibration mode of NEHs is usually much greater than that in high order vibration modes. Therefore, most of governing models of NEHs are simplified as the first-order model. For numerically simulating the nonlinear governing models of NEHs, the Runge-Kutta algorithm is a good option. However, the numerical solution highly depends on the initial conditions (initial displacement/velocity/voltage in simulation), and not all the solutions can be easily found [28]. This requires approximate analytic methods to theoretically predict multiple solutions of the model and provide a reference to numerically verify their existence and design optimal energy harvesting devices in experiments [29,30].

Nonlinear monostable vibration energy harvesters (1 equilibrium position) and bistable vibration energy harvesters (BEH: 2 stable and 1 unstable equilibrium positions) were widely investigated, and their broadband advantages over the linear counterpart were verified in both simulation and experiments [31,32]. Especially, BEHs have been widely investigated via different designs [33–36]. For example, mutually attractive or repulsive magnets which are respectively attached to the free end of a piezoelectric beam and fixed to the base structure can obtain a beam-based BEH [32,33]. In addition, a clamped-clamped piezoelectric beam which is initially buckled by applying a longitudinal compressive displacement or axial precompression can exhibit bistable characteristics [34]. Another method to make a BEH is employing bistable composite structures [35]. In detail, the thermal stresses of a composite laminate produced in the curing process can cause an out of plane curvature, and makes the composite laminate have two stable equilibrium positions [36]. If piezoelectric patches are attached to the surface of the composite laminate, a bistable composite energy harvester without any external load or magnet can be achieved [37,38].

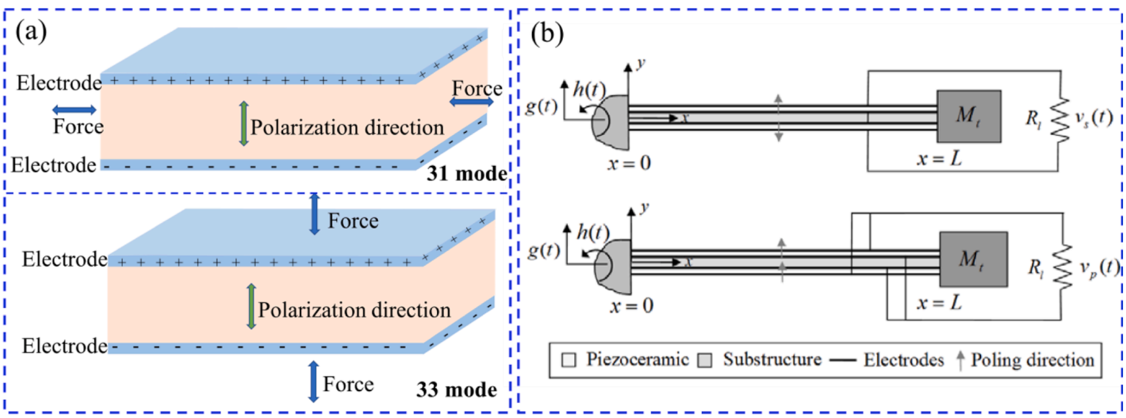

Fig. 1. (a) Illustration of commonly used operation modes for piezoelectric materials; (b) a cantilever-type harvester (Reproduced from [12], Copyright 2008, with the permission of IOP Publishing, Ltd).

Multistable vibration energy harvesters (MEH) have attracted increasing attention because of the excellent energy harvesting performance and interesting dynamic characteristics [39], as well as the tailoring opportunities they provide. The number of equilibrium positions of MEH is mathematically odd, and the number of stable equilibrium positions is always larger by a unit compared to unstable equilibrium positions. In physics, equilibrium positions correspond to zero points in the equivalent nonlinear restoring curve of MEH, as well as extreme points in the potential energy curve of the MEH, which also divide its motion into different ranges. For the MEH, intrawell oscillations owning small amplitude are confined in a potential well whose lowest point correspond to a stable equilibrium position. The motion range of high-energy interwell oscillations is across two or more potential wells, which lead to high power output.

This paper presents the principle, progress and perspectives of MEHs, and mainly focuses on the tristable vibration energy harvester (TEH). The broadband advantage in vibration energy harvesting, remarkable dynamic characteristics (as well as associated limitations) and challenging issues will be discussed. The organization of the paper is shown as follows: Governing models and approximate analytic methods are introduced in Section 2; Design, experiment, influence mechanism and optimization are illustrated in Section 3; In Section 4, we state the applications of MEHs in rotational motion, wind energy harvesting, etc, as application examples and case studies; Future challenges about MEHs are discussed in Section 5; Key conclusions and perspectives about future research are finally addressed at last.

2. Governing model and approximate analytic methods

To comprehensively investigate response characteristics of MEH, the governing model should be obtained which can provide a reference to the real performance. Usually, a dimensionless governing model provides the qualitative prediction without loss of generality, while the dimensional one gives quantitative output voltage and power which are critical to a particular design addressing a specific application case. The use of approximate analytic methods is to find all the possible solutions of the nonlinear model, and can consider the harmonic balance method, the method of multiple scales, the modified Lindstedt–Poincaré method, etc [7,23,32].

2.1. Governing model of the MEH

As shown in Fig. 2(a), Erturk et al [40,41]. designed a Duffing based BEH which is comprised of a ferromagnetic elastic beam (two piezoelectric patches are attached at the fixed end), an electric load and two magnets fixed to a supporting structure. Because of the magnetic action, the proposed harvester owns a wider working frequency range than the linear counterpart. Two dimensionless electromechanical equations were presented for predicting output characteristics of the harvester [40], as follows:

$$\ddot{x} + 2\zeta\dot{x} - \frac{1}{2}x\left(1 - x^2\right) - \chi v = f\cos\Omega t \tag{1}$$

$$\dot{v} + \lambda v + \kappa\dot{x} = 0 \tag{2}$$

where v is the output voltage across the load resistance R_l. χ is the dimensionless piezoelectric coupling coefficient. κ is the dimensionless piezoelectric coupling term. λ is the reciprocal of the dimensionless time constant ($\lambda\infty 1/R_l C_p$ where C_p is the equivalent capacitance of piezoelectric patches). It is noted that parameters in Eqs. (1) and (2) are dimensionless and the numerical results are dimensionless, which can provide qualitative results guiding the design of BEHs.

In 2013, Zhou et al. [42] designed a rotatable NEH by changing the inclination angle of two external magnets to further extend its working frequency range, as shown in Fig. 2(b). Different with the horizontally placed ones, the two external magnets have an inclination angle α with the horizontal direction, and different α gives the harvester different nonlinear characteristics leading to the

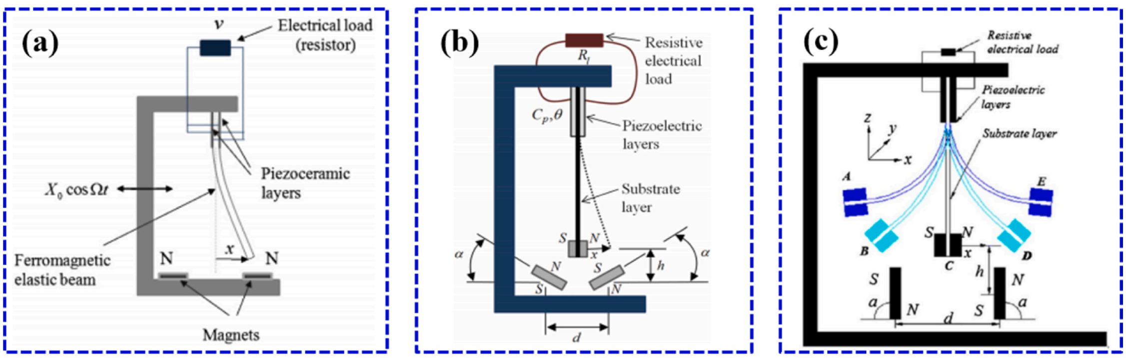

Fig. 2. (a) A Duffing based BEH (Reproduced from [41], Copyright 2011, with the permission of Elsevier); (b) a rotatable NEH (Reproduced from [42], Copyright 2013, with the permission of AIP Publishing); (c) a typical TEH (Reproduced from [27], Copyright 2014, with the permission of Elsevier).

wider working frequency range. In addition, two tip magnets bring a large nonlinear magnetic force to the harvester. This design saves the real physical space, and nonlinear monostable/bistable/multistable vibration energy harvesters can be obtained based on this design. A quantitative governing model was presented, as described by Eqs. (3) and (4) [42]:

$$m\ddot{x} + c\dot{x} + kx - \theta v = F + F_m \tag{3}$$

$$C_p\dot{v} + \frac{v}{R_l} + \theta\dot{x} = 0 \tag{4}$$

where m, c and k are the equivalent mass, equivalent damping and equivalent stiffness of the harvester, respectively. F can be considered as an external mechanical force as an excitation term. v and x respectively denote the output voltage and tip displacement relative to the base. θ is the equivalent electromechanical coupling coefficient. F_m is the magnetic force measured in experiment by a micro dynamometer and fitted by a polynomial function. Note that we take the piezoelectric case as an example to explain the model, and harvesters with the electromagnetic, electrostatic or triboelectric energy converter have a similar model with different electromechanical coupling.

Based on the BEH design depicted in Fig. 2(b), a TEH was derived as shown in Fig. 2(c), which has five equilibrium positions (**B** and **D** are unstable equilibrium positions; **A, C** and **E** are stable equilibrium positions). If F_m moves from the right side of Eq. (3) to its left side, it is possible to get the equivalent nonlinear restoring force F_{non}=kx - F_m. In addition, F can be replaced by a special equivalent base excitation term $\eta m \cdot (-\ddot{x}_b)$ in the case of harmonic base excitation Eq. (3). will thus be written as:

$$m\ddot{x} + c\dot{x} + F_{non} - \theta v = \eta m \cdot \left(- \ddot{x}_b \right) \tag{5}$$

where x_b is the base displacement excitation. μ is the amplitude-wise correction factor for the lumped parameter model (greater or equal to 1) [2]. A polynomial function can be to describe F_{non}, as follows [43]:

$$F_{non} = m_0 + m_1x + m_2x^2 + \ldots + m_nx^n \tag{6}$$

where m_0, m_1, m_2, . . ., m_n are polynomial coefficients, which determine the number of equilibrium positions of the harvester.

Deriving F_{non} yields the equivalent nonlinear stiffness which may visually show the nonlinear nature of the harvester. The equivalent schematic diagram of the general NEH (with piezoelectric/electromagnetic/electrostatic/triboelectric materials, etc.) subjected to the base excitation x_b is drawn in Fig. 3.

The experimental device and three stable equilibrium positions of a TEH are shown in Fig. 4(a) [43]. As mentioned before, the nonlinear magnetic force is very complex and it is difficult to get exact theoretical results for the cantilever type NEH. However, polynomial fit can be used with respect to experimentally measured data of the nonlinear magnetic force to obtain a simplified expression, as shown in Fig. 4(b). Equilibrium positions correspond to zero points in the equivalent nonlinear restoring force curve, and maxima and minima of the potential energy curve. Based on the Runge-Kutta algorithm, numerical response displacement and output

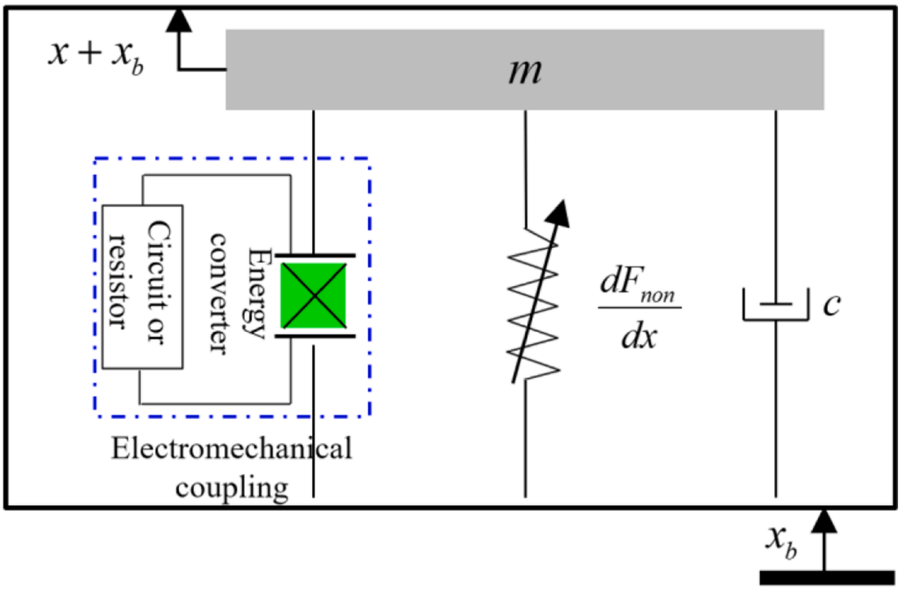

Fig. 3. The equivalent schematic diagram of the general NEH.

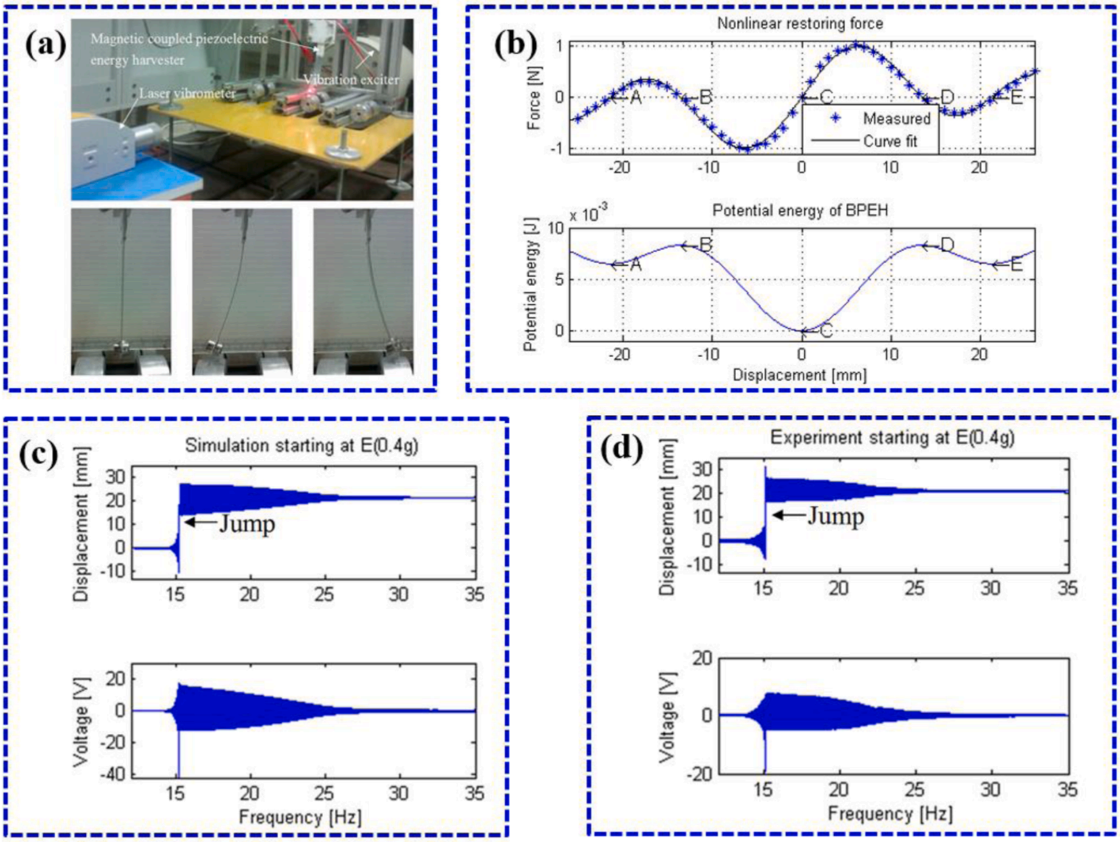

Fig. 4. The TEH: (a) Experimental setup and three stable equilibrium positions; (b) equivalent nonlinear restoring force and potential energy curves; (c) numerical output voltage; (d) experimental output voltage (Reproduced from [43], Copyright 2014, with the permission of The European Physical Journal).

voltage of the governing model can be obtained (Eqs. (4) and (5)), which match well with experimental results, as illustrated in Fig. 4 (c) and (d).

As an alternative, calculation models for the nonlinear magnetic force were also explored [44], while the simplified models cannot reveal the exact relative position between the large-deformation beam and external magnets, which inevitably leads to calculation error [45–47]. Since the magnetic force is very sensitive to the relative position of magnets, how to accurately obtain the relative position of the tip magnet attached on the free end of the beam and external magnets is vital. However, the beam structure has geometric nonlinear deformation and non-negligible axial displacement, which are not fully considered in the current calculation models for the nonlinear magnetic force. In other words, it is very difficult to get the relative position of magnets based on current methods. This inevitably brings errors for predicting the nonlinear magnetic force. Nonlinear finite element method and other complex methods may be a solution for more accurately calculating any relative position of magnets thus improving the calculation accuracy of the nonlinear magnetic force [48,49]. Note that it is not necessary to solve the time-domain governing model of the harvester for calculating the nonlinear magnetic force, as only the static relative positions of magnets are necessary. Complex methods would however yield intensive calculation process and are time consuming. Therefore, a trade-off should be met to obtain a suitable method to calculate the nonlinear magnetic force. After getting all the parameters, the Runge-Kutta algorithm, the Newton iteration method, etc., are typical approaches to get the numerical response displacement (including chaotic vibrations) and output voltage of the governing model Eqs. (4) and (5).

2.2. Approximate analytic methods

Under harmonic excitation, $-\ddot{x}_b$ in Eq. (5) can be replaced by $A\cos(\omega t)$ (A and ω respectively being the base acceleration excitation amplitude and angular frequency). We all know that, a MEH may have several vibration orbits corresponding to different solutions of their governing model in some excitation conditions, while numerical solutions highly depend on initial conditions at the beginning of simulation. Therefore, approximate analytic methods are used for finding all the solutions and providing an analytical framework for experiments and numerical simulations.

Taking the harmonic balance method as an example (by assuming a single harmonic), if we assume that the displacement response and the output voltage of MEHs have slowly varying coefficients (a_0, a and b) [50,51], as follows:

$$x = a_0 + a\sin(\omega t) + b\cos(\omega t) \tag{7}$$

$$\dot{x} = \dot{a}_0 + (\dot{a} - b\omega)\sin(\omega t) + (\dot{b} + a\omega)\cos(\omega t) \tag{8}$$

$$\ddot{x} = -(2\dot{b} + a\omega)\omega\sin(\omega t) + (2\dot{a} - b\omega)\omega\cos(\omega t) \tag{9}$$

$$v = d\sin(\omega t) + e\cos(\omega t) \tag{10}$$

$$\dot{v} = (\dot{d} - e\omega)\sin(\omega t) + (\dot{e} + d\omega)\cos(\omega t) \tag{11}$$

We can substitute Eqs. (7)–(11) into Eqs. (4) and (5), and balance the terms with $\sin(\omega t)$ and $\cos(\omega t)$ as well as constant terms, the analytical solutions can be achieved by solving the coupled equations [50,51]. The output voltage amplitude and the displacement amplitude are respectively $V = \sqrt{d^2 + e^2}$ and $r = \sqrt{a^2 + b^2}$. The relationship between V and r is given as:

$$V = \left| \frac{\theta\omega}{\sqrt{\frac{1}{R_l^2} + (C_p\omega)^2}} \right| \cdot r \tag{12}$$

The stability of solutions is determined by the following Jacobian matrix:

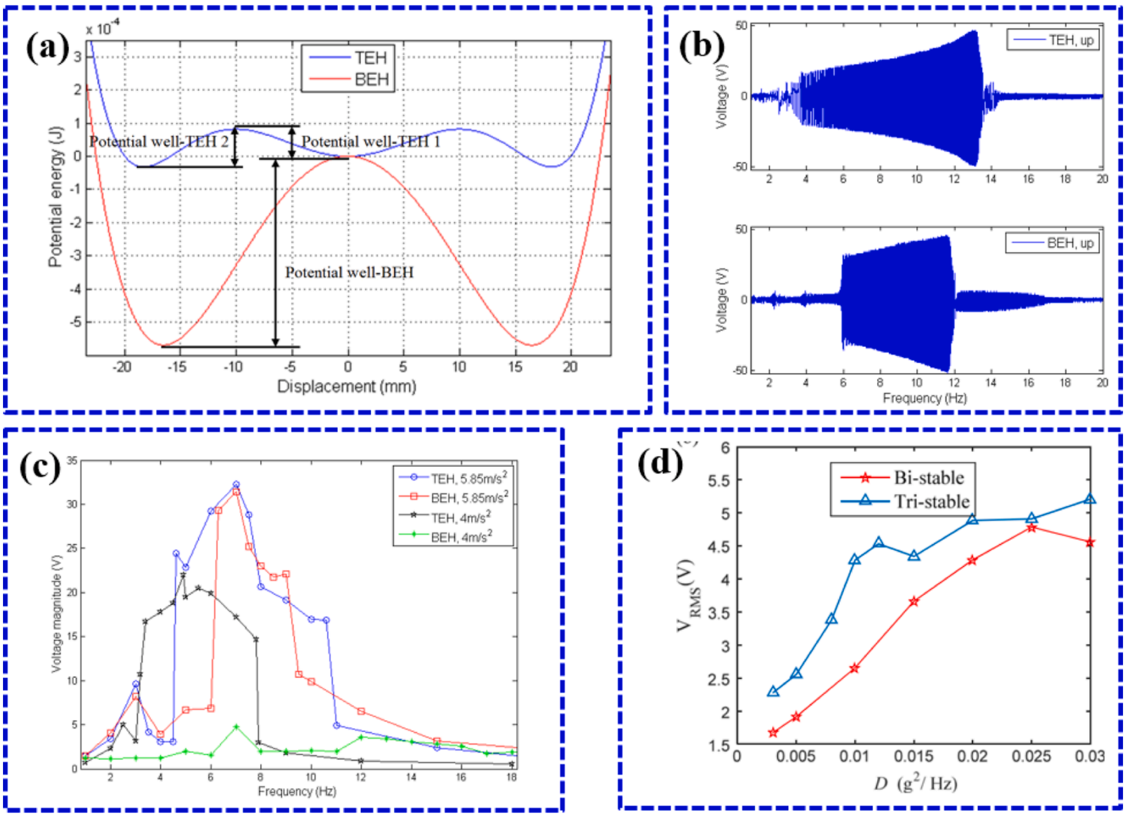

Fig. 5. Comparison of the BEH and TEH: (a) Potential energy curves (Reproduced from [27], Copyright 2014, with the permission of Elsevier); (b) output voltages of harvesters under frequency-swept harmonic excitations (Reproduced from [27], Copyright 2014, with the permission of Elsevier); (c) output voltages of harvesters under constant frequency harmonic excitations (Reproduced from [27], Copyright 2014, with the permission of Elsevier); (d) RMS output voltages of harvesters under stochastic excitations (Reproduced from [61], Copyright 2016, with the permission of IOP Publishing, Ltd).

$$J = \begin{bmatrix} \frac{\partial \dot{a}_0}{\partial a_0}, \frac{\partial \dot{a}_0}{\partial a}, \frac{\partial \dot{a}_0}{\partial b}, \frac{\partial \dot{a}_0}{\partial d}, \frac{\partial \dot{a}_0}{\partial e} \\ \frac{\partial \dot{a}}{\partial a_0}, \frac{\partial \dot{a}}{\partial a}, \frac{\partial \dot{a}}{\partial b}, \frac{\partial \dot{a}}{\partial d}, \frac{\partial \dot{a}}{\partial e} \\ \frac{\partial \dot{b}}{\partial a_0}, \frac{\partial \dot{b}}{\partial a}, \frac{\partial \dot{b}}{\partial b}, \frac{\partial \dot{b}}{\partial d}, \frac{\partial \dot{b}}{\partial e} \\ \frac{\partial \dot{d}}{\partial a_0}, \frac{\partial \dot{d}}{\partial a}, \frac{\partial \dot{d}}{\partial b}, \frac{\partial \dot{d}}{\partial d}, \frac{\partial \dot{d}}{\partial e} \\ \frac{\partial \dot{e}}{\partial a_0}, \frac{\partial \dot{e}}{\partial a}, \frac{\partial \dot{e}}{\partial b}, \frac{\partial \dot{e}}{\partial d}, \frac{\partial \dot{e}}{\partial e} \end{bmatrix} \tag{13}$$

Extending the solution to other harmonics (including sub-harmonics and super-harmonics [30,52,53]) besides the fundamental harmonic may be achieved by including terms in Eqs. (7)–(11) and performing a similar solving process. In addition, the method of multiple scales, the modified Lindstedt–Poincaré method, the complexification-averaging method, etc., can be also used to get approximate solutions of MEHs under the harmonic excitation [54–56]. For predicting the chaotic motion of nonlinear systems, the Melnikov method and the Shornikov method are a good choice [57,58]. For nonlinear systems under random excitations, analytical and semi-analytical methods are usually employed, such as stochastic averaging method, Fokker-Planck-Kolmogorov equation, etc. [59].

3. Energy harvesting performance and enhancement strategy

In the last section, Eqs. (1)–(13) established the governing model and its general approximate solutions which can be used to predict response characteristics and the output voltage/power. However, the real performance of MEHs depends on the physical design and ambient environments. This section reviews several designs and experimental assessments of MEHs, and summarizes the influencing mechanism and optimizing strategies for improving the vibration energy harvesting performance.

3.1. Design and experimental test

The design of MEHs can be achieved via magnetic attraction/repulsion, magnetic levitation, geometric nonlinearity and so on. For instance, a typical design of a TEH based on magnetic attraction/repulsion is shown in Fig. 2(c), and it is found that the TEH can be designed to have lower potential barriers than a BEH with higher potential barriers as shown in Fig. 5(a). From the view of physics, the

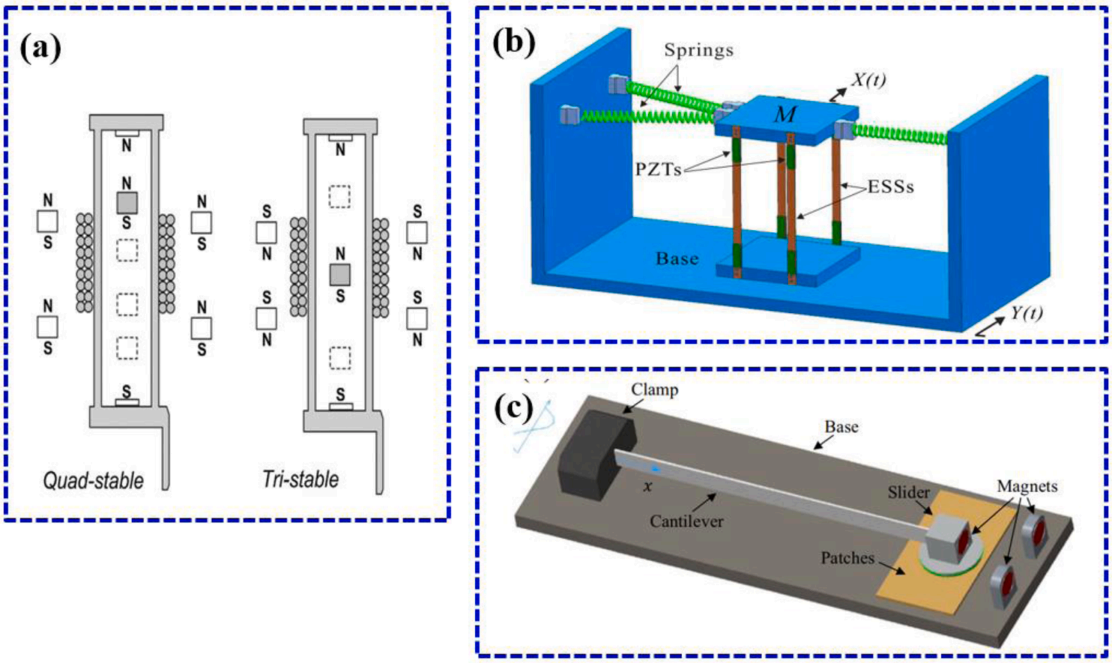

Fig. 6. (a) A multistable electromagnetic-induction energy harvester (Reproduced from [70], Copyright 2018, with the permission of Elsevier); (b) a MEH based on geometric nonlinearity (Reproduced from [71], Copyright 2019, with the permission of IOP Publishing, Ltd); (c) a TEH with triboelectric material (Reproduced from [72], Copyright 2020, with the permission of Springer Nature (the Creative Commons CC BY license)).

former can more easily cross potential wells and realize high-energy interwell oscillations leading to high output power under low-level excitations. Another option denoting the adaptability of the TEH over the BEH is keeping the same potential barrier, which may lead to larger vibration amplitude for the TEH. Experimental results in Fig. 5(b) and (c) verify that the TEH owning shallower potential wells has better energy harvesting performance than the BEH under both frequency-swept and constant frequency low-level harmonic excitations. The same conclusion was also addressed by Zhu et al. [60]. Based on another TEH implementing magnetic attraction/repulsion, Li et al. [61] experimentally found that the TEH generated larger RMS output voltage than the BEH under low-intensity stochastic excitations as shown in Fig. 5(d). Under a filtered Gaussian noise excitation within the frequencies ranging from 0 to 120 Hz, Leng et al. [62] also verified the better energy harvesting performance of the TEH over the BEH in experiment. Zhang et al. [63] used a linear-arch composite beam to replace the straight rectangular beam of the TEH, and explored the effect of the magnet distance on the potential curves and dynamic responses. By adding more external magnets to the cantilever-type bistable or triable configuration, Zhou et al. experimentally studied quad-stable [64] and penta-stable [65] vibration energy harvesters and checked the unique performance. Hence, taking advantage of increasing the number of potential wells, all of these studies demonstrated several means for achieving easier interwell motion and therefore higher energy harvesting performance.

Zayed et al. [66] designed a 2-DOF quad-stable vibration energy harvester, which is composed of an inner beam with a tip magnet, a hollow outer beam and three fixed external magnets. They set 4 V as the targeted output voltage, and the harvester was connected with a resistive load. In this case, the harvester has an effective working bandwidth wider than 7.3 Hz at the excitation level of 3 m/s^2 in experiments. It should be noted that the bandwidth of NEHs (monostable, bistable, multistable vibration energy harvesters) connected with a resistive load usually depends on the hypothetical conditions, such as oscillation at the highest orbit, targeted output voltage, etc. [27,32,41,66,67]. For the harvester connected with a circuit, the definition of bandwidth is often related to the voltage (or power) response or the response spectral [68,69]. Therefore, there has been no scientific consensus about how to define the effective working bandwidth of vibration energy harvesters by far.

Different with the magnetic attraction/repulsion, the magnetic levitation can be used to form multistable configurations, while the size of the device is directly proportional with moving magnets and range of motion. Gao et al. [70] designed a multistable electromagnetic-induction energy harvester via the magnetic levitation (Fig. 6(a)), which is comprised of a tube, a moving magnet and two fixed magnets inside the tube, several outer static magnets, coils installed between the moving and static magnets. The number of equilibrium positions mainly depends on the number of outer static magnets. In experiments, they set the base acceleration excitation as 2 g and connected the harvester with a single load resistance of 41.5 Ω. The harvester has a total length of 330 mm and maximum diameter of 150 mm and generates RMS current of 80.15 mA and RMS power output of 440.98 mW in the wide frequency range of 5–12 Hz.

Based on the geometric nonlinearity, Yang and Cao [71] presented a MEH based on geometric nonlinearity (Fig. 6(b)), which is comprised of four symmetrically assembled beams, one spring in the horizontal direction, two springs with an inclination angle and one mass block in the center. By changing the geometric parameters, the harvester may exhibit monostable, bistable, tristable and

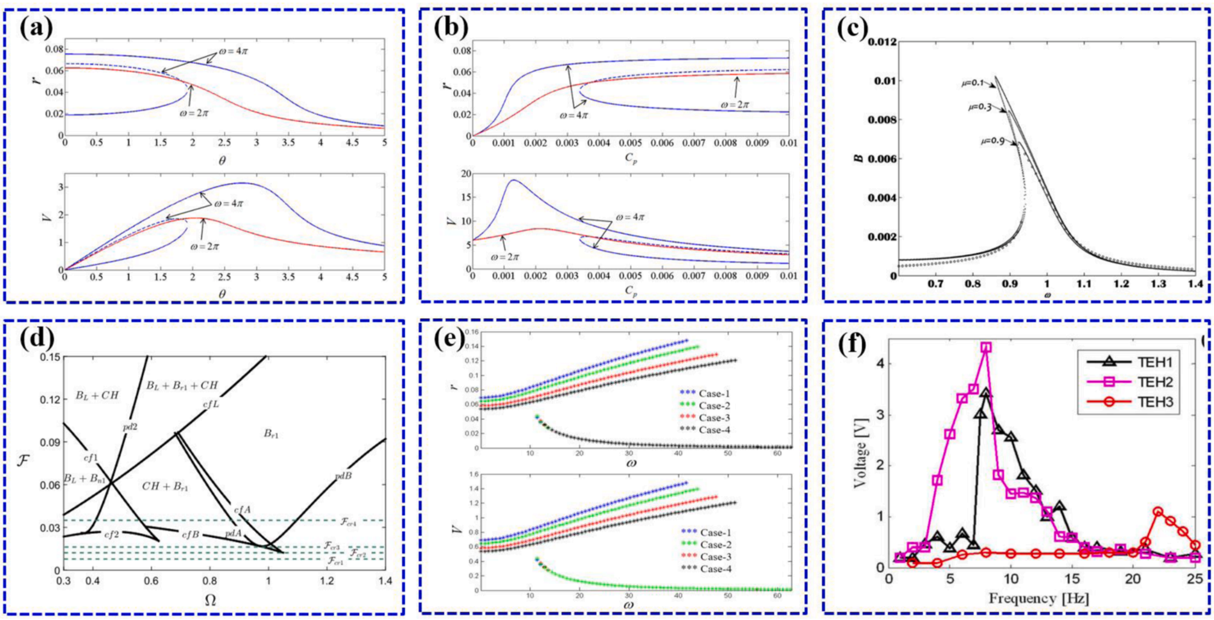

Fig. 7. (a) and (b) Influence of optimized electromechanical coupling coefficient and capacitance, respectively (Reproduced from [50], Copyright 2016, with the permission of Elsevier); (c) influence of the viscoelasticity coefficient (Reproduced from [78], Copyright 2015, with the permission of AIP Publishing); (d) bifurcation map (Reproduced from [54], Copyright 2017, with the permission of Elsevier); (e) influence of asymmetry of potential wells (Reproduced from [51], Copyright 2018, with the permission of Elsevier); (f) experimental response of TEHs with different potential wells (Reproduced from [79], Copyright 2015, with the permission of AIP Publishing).

quad-stable characteristics, which can be used to improve the energy harvesting performance from time-varying ambient vibrations. Fu et al. [72] designed a sliding-mode triboelectric TEH, as shown in Fig. 6(c). When the cantilever beam vibrates, the slider attached in its free end will contact with triboelectric material thus generate electrical charge. After deducing a theoretical model of the TEH connected with a load resistance, the simulations demonstrate that as the increasing of friction, the frequency range of interwell oscillations will decrease and the energy harvesting efficiency will inevitably reduce.

3.2. Influence mechanism and optimization

As a highly nonlinear system, the complex response characteristics of the MEH were numerically and experimentally tested. The changing of stable and unstable equilibrium positions could be found from bifurcation diagrams of the equilibrium solutions versus geometrical parameters of the harvester [73,74]. It is found that intrawell and interwell oscillations are determined by system parameters of the harvester and excitation conditions [75–77].

To analytically reveal the influence mechanism, Zhou et al. [50] derived analytic solutions of the TEH based on the harmonic balance method for revealing the nonlinear response mechanism and predicting the output voltage. An important finding is that the optimized electromechanical coupling coefficient (Fig. 7(a)) and capacitance (Fig. 7(b)) are relevant to excitation conditions. The electromechanical coupling will also bring equivalent electrical damping which influences the harvester's high-energy interwell oscillations, therefore, a larger electromechanical coupling coefficient is not always better. Tékam et al. [78] used the Krylov-Bogolyubov averaging method to get the approximate solutions of a TEH with the fractional order viscoelastic material, and the order of fractional derivative was found to have an obvious influence on the effective frequency range and the threshold curve of horseshoes chaos, as shown in Fig. 7(c). However, how to fabricate a harvester with the fractional order damping in the practical engineering is worthy of consideration. Panyam and Daqaq [54] employed the method of multiple scales to approximate the dynamic behavior of the TEH, and they used the bifurcation map to identify and characterize boundaries of intrawell and interwell oscillations in the force-frequency parameter space (Fig. 7(d)).

Zhou and Zuo [51] found that the influence mechanism of the asymmetric feature in potential wells on the output voltage allows changing potential wells and adjusting the distribution of potential energy. In this case, the potential barriers are inevitably changed, which significantly influence the amplitude of the interwell oscillation and the working frequency range, as shown in Fig. 7(e). Under harmonic excitations, Cao et al. [79] (Fig. 7(f)) and Kim et al. [80] respectively experimentally verified that the threshold value of the

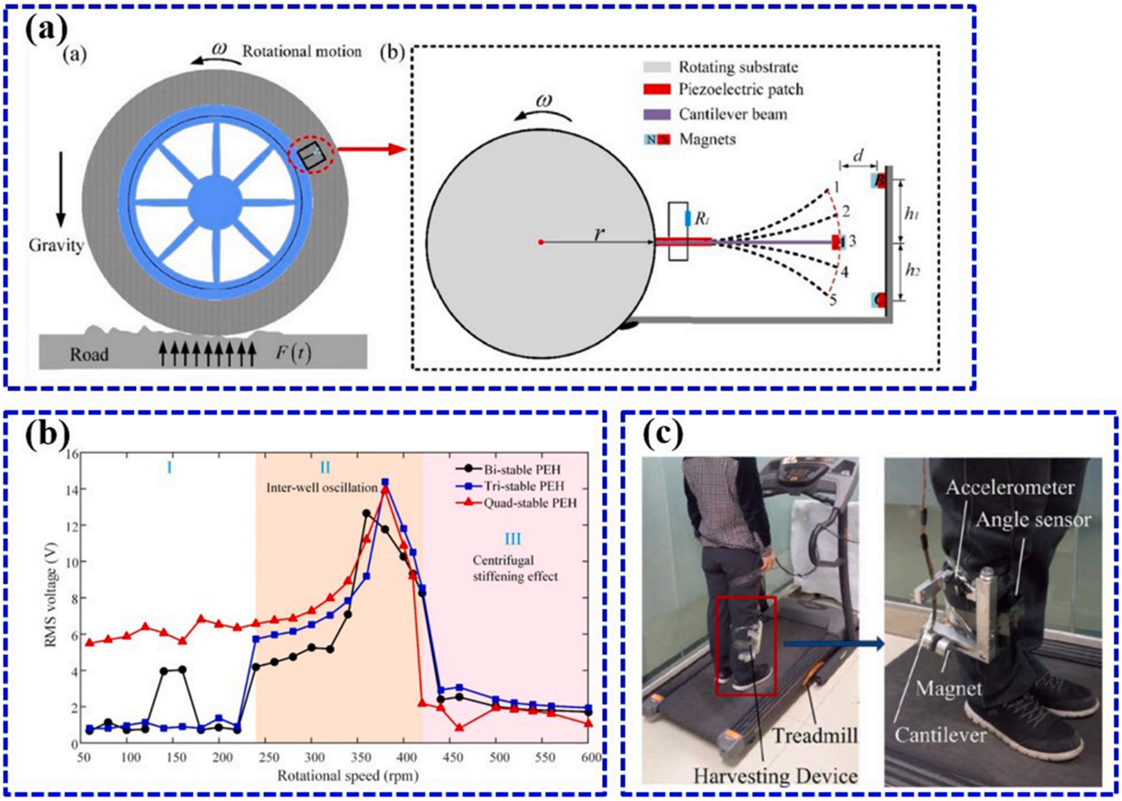

Fig. 8. (a) A TEH installed on a vehicle wheel (Reproduced from [92], Copyright 2020, with the permission of Elsevier); (b) RMS output voltages under a wide rotational speed range (Reproduced from [93], Copyright 2021, with the permission of Elsevier); (c) a TEH bundled on the lower leg (Reproduced from [95], Copyright 2017, with the permission of Elsevier).

base excitation amplitude for interwell oscillations is increased along with the increasing of the potential well depth or the potential barrier height. Therefore, if we want to get interwell oscillations and improved energy harvesting performance from low-level vibrations, the potential barrier height should be short, and vice versa. In the same physical space, Lallart et al. [81] found that the increasing number of equilibrium positions will decrease the potential barrier height, and this makes the MEH more easily enter into high-energy interwell oscillations and own a wider working frequency range under low-level excitations. Another option revealed by this work is to keep the same potential barrier, which would yield much larger displacement magnitude, thus increasing the output power. Meanwhile, the height of the interwell orbit will decrease, thus the output voltage from interwell oscillations will also decrease. In addition, the approximate solutions of the MEH with high-order stiffness terms (this also leads more equilibrium positions) can be found in Ref. [55].

It is well known that pure harmonic excitation is ideal, while there are a lot of random excitations in ambient environment which affect the energy harvesting performance of BEHs [82,83]. Zhang et al [84,85]. analyzed the stochastic bifurcations and the response characteristics of a TEH subjected to colored noise, and they obtained the averaged Fokker–Plank–Kolmogorov (FPK) equation and the stationary probability density of the amplitude via the stochastic averaging method. In addition, Monte Carlo simulations were selected for verifying the theoretical results. The potential function of the TEH should be carefully designed to match with the colored noise [86]. To reveal the resonance mechanism of the MEH under stochastic parametric excitations, Huang et al. [87] employed the finite difference method to work out the FPK problem which is related with two-dimensional Itô stochastic differential equations. They found that the number of stable equilibrium positions influences the trivial and nontrivial steady-state solutions. Wang et al. [88] investigated stochastic characteristics of asymmetric MEHs and provided an approximate FPK equation. They found that at a constant noise intensity, the probability distribution of velocity response of the harvester is almost unaffected by the potential function shape.

4. Design for applications

The above investigations focused on the performance of MEHs under base excitations and provide fundamentally theoretical and experimental framework. In the real engineering practice, excitations may be however much more complex. For example, rotational motions are widely existing in ambient environments, such as rotor, vehicle wheel, gearbox, human motion, etc. [10,89,90].

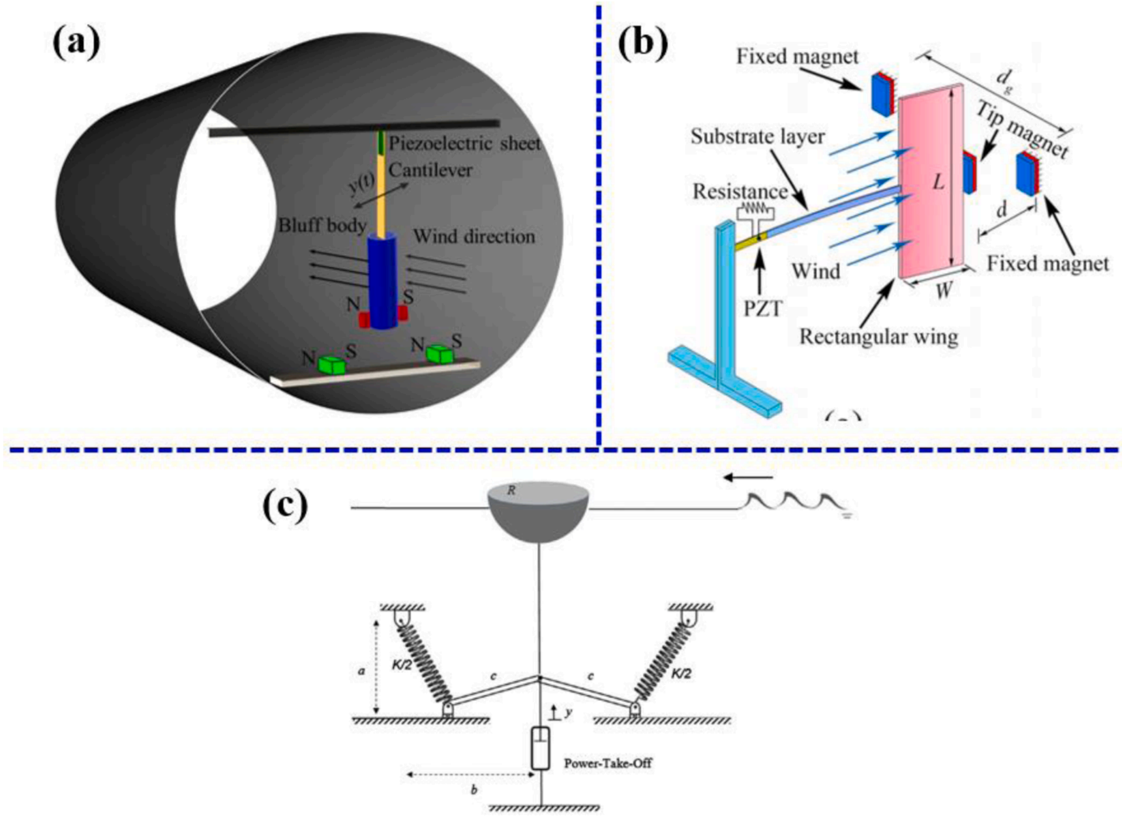

Fig. 9. (a) A vortex-induced vibrational TEH (Reproduced from [100], Copyright 2019, with the permission of IOP Publishing, Ltd (the Creative Commons Attribution 3.0 licence)); (b) dynamic-stable flutter energy harvester (Reproduced from [102], Copyright 2019, with the permission of AIP Publishing); (c) a multistable ocean wave energy harvesting system (Reproduced from [103], Copyright 2017, with the permission of Elsevier).

4.1. MEH in rotational motion

Unique designs based on the multistable mechanism were investigated to realize effective rotational energy harvesting. Mei et al [91,92]. proposed the idea that MEH can be installed on a vehicle wheel and harvest mechanical energy from the wheel's rotational motion through the vibration of the harvester, with the modeling schematic diagram shown in Fig. 8(a). By using the Lagrange equation, the nonlinear magnetic force calculation model and so on, they gave a theoretical model to predict nonlinear dynamic response characteristics and output voltages of the MEH [92]. It was found that bistable/tristable/quad-stable vibration energy harvesters could vibrate in their high-energy interwell orbit and generate large-amplitude output voltage under appropriate rotation conditions [93], as shown in Fig. 8(b). During the human walking or running, there is also a circular/rotational motion around the knee. Donelan et al. [94] designed an electromagnetic energy harvester which is used to harvest mechanical from human walking via the rotation of the generator. Different with this design, Wang et al. [95] installed a piezoelectric TEH bundled on the lower leg, which generated electricity during human walking based nonlinear vibration of the TEH (Fig. 8(c)). Broadband characteristics and high-performance energy harvesting were numerically and experimentally verified.

Here, we aim at analyzing the centrifugal effect and design of MEHs in rotational motion. If the free end of a piezoelectric beam is away from the center of the rotational plate (such as that in Fig. 8(a)) [92,96], the centrifugal force will pull the piezoelectric beam in the axial direction. With the continuous rotation of the wheel, the centrifugal force modifies the stiffness and resonant frequency of the piezoelectric beam which becomes a centrifugal hardening beam. Along with the increasing of the rotational speed/frequency, the resonant frequency of the piezoelectric beam may be self-tuned and match with the rotational motion. If the free end of a piezoelectric beam is towards the center of the rotational plate, the centrifugal force will work as an axial compressive force to the piezoelectric beam which becomes a centrifugal softening beam [97]. Fang et al. [97] used the centrifugal softening effect to enhance the performance of a rotational impact energy harvester. Therefore, how to take advantage from the rotational motion depends on the structural design and electrical boundary conditions of a MEH.

4.2. MEH in wind energy harvesting

Besides rotational motion, wind energy widely exists in open country, high buildings and ventilating ducts, and is inexhaustible and environment-friendly energy source. Because of the large size and high cut-in wind speed, traditional windmill power generators are not suitable for powering wireless sensors under weak wind environment. To solve this issue, wind-induced vibration energy harvesters using vortex-induced vibration (VIV), galloping and flutter mechanisms may effectively harvest wind energy at a small scale [98,99]. To further enhance the wind energy harvesting performance, Zhou et al. [100] brought the multistable mechanism to the design of wind energy harvesters, and the schematic diagram of the vortex-induced vibrational TEH is shown in Fig. 9(a). Wang et al. [101] numerically and experimentally found that a broadband tristable galloping piezoelectric energy harvester with the optimal design can realize high-energy oscillations and exhibit larger output voltage than a traditional galloping energy harvester at some wind speeds. Zhou et al. [102] investigated a dynamic-stable flutter energy harvester with a rectangular win as shown in Fig. 9(b), and they experimental found that the harvester can perform snap-through motions and output large voltage at different wind speeds.

In addition, Younesian and Alam [103] proposed a multistable ocean wave energy harvesting system which consists of a hemispheric buoy and a power take off system as shown in Fig. 9(c), and gave a dimensionless governing model of this system. High-energy interwell oscillations and chaotic motions were numerically found for this system, which may be beneficial for energy harvesting. However, test in real oceanic conditions is necessary to check the energy harvesting performance and efficiency of the multistable ocean wave energy harvesting system. For large-scale energy harvesting in real-world wave excitation, various non-resonating energy harvesting systems with large output power were designed and tested [104]. For example, Liang et al. [105] designed a non-resonating wave energy converter containing a buoy (1.2 m) and a mechanical motion rectifier based power takeoff system. The peak output power reached 205 W and the average power was 21 W in real oceanic test (the wave height was 0.2 m and the dominate wave period was 4 s). In the future, the design and experimental tests of ocean wave energy harvesters with multistable characteristics for specific output power scale may be of particular interest.

Tan et al. [106] proposed a quad-stable wake-galloping energy harvester which is a magnetic coupled quad-stable piezoelectric beam and a fixed square rod. The influence of the magnet distance on the potential energy function, the energy harvesting performance was explored, and it was found that the larger magnet distance makes the harvester more easily to cross over the barrier leading to large-amplitude output voltage. Li et al. [107] analytically studied a piezoelectric MEH under wake-galloping via the Melnikov method. In detail, they obtained chaos criterion based on the changing of the homoclinic-like and heteroclinic-like orbits of the harvester with viscous damping and an external wake-galloping.

4.3. MEH in structural health monitoring and vibration suppression

Vibration energy harvesting technique is expected to be a relatively new method for supplying sustainable energy to low-powered sensor networks, which plays a key role in structural health monitoring. In some cases, the output voltage of harvesters can be considered as the signal which reflects the structural health status of the host structure. Furthermore, energy harvesting can address the issue of wirings in such cases (cost, weight, reliability…), especially in transportation (aeronautic or railway applications for instance). Elahi [108] proposed a piezoelectric energy harvester to simultaneously harvest energy from flow-induced structural vibrations and performing structural health monitoring. In simulations, they compared obtained susceptance value with the initial value of a pristine system to determine if there was structural damage. However, the real performance and feasibility should be verified in experiments.

Fitzgerald et al. [109] used cantilever-based piezoelectric vibration energy harvester connected with a load resistance to measure bridge frequency shifts arising due to scour, and they verified the feasibility in experiments. Although there is no similar report on MEHs, their output voltage can be also used for structural health monitoring in some cases. MEHs are sensitive to ambient vibrations, which may be useful for detecting weak vibrations in the future.

From the viewpoint of conservation of energy, when the harvester extracts vibration energy from the host structure, it also reduces the vibration of the latter, and thus, in a complimentary and upstream way of SHM, provides means of potentially limiting damage occurrence. Yang et al. [7] presented a comprehensive review on vibration energy harvesting and vibration suppression technologies, and discussed the potential applications in aerospace/marine/transport engineering and health monitoring and vibration control etc. Huang and Yang [110] made numerical exploration on combination of vibration absorption and energy conversion of MEH via a dynamical model of a 2-dof dynamic vibration absorber. They found that a tristable electromagnetic energy harvester could induce a rapid attenuation in the residual kinetic energy of the primary system (excited by an impulse) thus realizing vibration suppression. However, the experimental verification has not been reported by far. In the future, it is of interest to combine active/ passive control strategy and multistable vibration energy harvesting for vibration suppression/control of a specific host structure.

5. Challenges

Although unique advantages of MEHs were assessed, there are still several research directions worthy of investigating to deeply reveal their nature. This section discusses four critical challenges.

5.1. Influence mechanism of external circuit

The vibration energy harvester usually generates an AC voltage under environmental excitations, however, wireless sensors and portable devices in many applications need a DC voltage. Therefore, an electrical interface between the energy converter and the

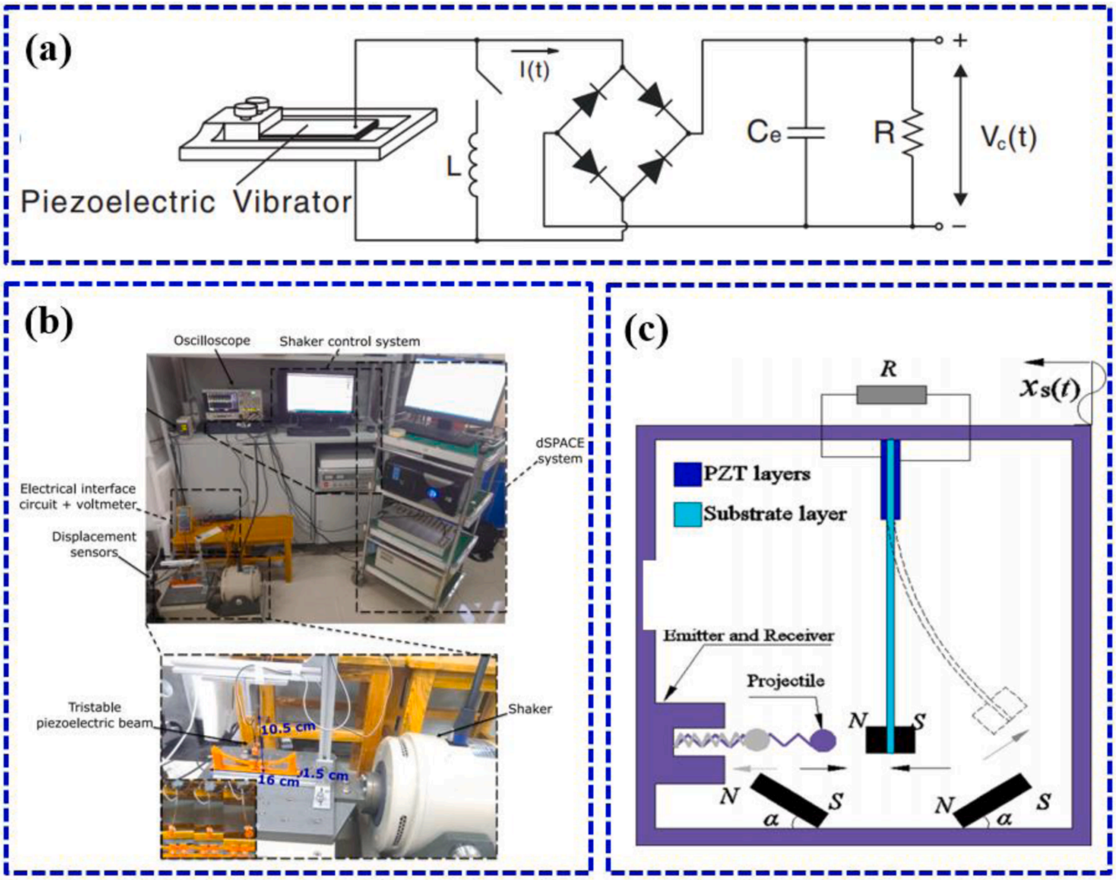

Fig. 10. (a) The SSHI electronical interface (Reproduced from [114], Copyright 2007, with the permission of IOP Publishing, Ltd); (b) the effect of synchronized discharging on TEHs in experiments (Reproduced from [119], Copyright 2020, with the permission of Elsevier); (c) an impact-based structure for realizing high-energy global oscillations under low-level excitations (Reproduced from [122], Copyright 2015, with the permission of AIP Publishing).

terminal powered device or the energy storage battery is essential. Additionally, electrical interfaces may also be taken into advantage for enhancing the electromechanical energy conversion performance of weakly coupled transducers, such as the synchronized switch harvesting on inductor (SSHI) or the Synchronized Electric Charge Extraction approaches [111–113]. Shu et al. [114] analyzed the performance of a piezoelectric energy harvester connected with a SSHI electronic interface, as shown in Fig. 10(a), and they found the good adaptability of the SSHI for either weakly or strongly coupled electromechanical system. Chen et al. respectively explored an improved parallel synchronized switch harvesting on inductor (P-SSHI) circuit with controllable optimal voltage (COV-PSSHI) [115], and an improved self-powered parallel SSHI [116] to maintain high conversion efficiency of NEHs, and the maximum average output power can be increased. Yan et al. [117] connected a series resistor-inductor resonant (RL) circuit to the piezoelectric patches of a TEH. They found the resonant frequency of the RL circuit greatly influences the output voltage amplitude of the TEH. Later on, Huang et al [56,118]. presented complete approximate analytic solutions of the TEH with a RL circuit, and the critical boundary of the multi-solution region can be calculated. Lallart et al. [119] experimentally explored the effect of synchronized discharging on the TEH, and they found the backward coupling yields degraded performance lightly damped structures on the bandwidth in the highly coupled case, as the experimental setup and structure shown in Fig. 10(b). However, this work also revealed the possibility for controlling the trade-off between effective frequency range and power boosting effect through the phase delay between voltage extremum occurrence and energy extraction event.

By far, most researchers focus on the structural design and broadband characteristics of NEHs. There is still few research which completely reveals the influence mechanism of the electrical interface or the external circuit on the dynamic response of NEHs, which impedes the effective optimization design and the real application. In the future research, we may pay attention to the coupling between the external circuit and the energy harvesting structure, and find an optimal design strategy.

5.2. Realizing the high-energy orbits

It is well known that NEHs have several vibration orbits and hysteresis region under some environment excitations especially for low-frequency high-level excitations. The broadband advantage can be realized with the precondition that they should work in the high-energy orbit of the multi-solution range. Under zero initial conditions, NEHs usually vibrate at the low-energy orbit, which leads to small-amplitude output voltage.

If we control the initial conditions to appropriate ones by using mechanical or electrical method [120–122], NEHs can be induced into the highest orbit which leads to high-efficient energy harvesting under the same excitation. In 2015, Zhou et al. [122] designed an impact-based structure for inducing NEHs to break the limitation from potential barriers and realizing high-energy global oscillations subject to low-level excitations, as shown in Fig. 10(c). In experiments, the effective frequency range of the BEH and the TEH was respectively increased by 467% and 56% under an excitation level (0.35 g). Based on adaptive structural designs, passive self-tuning energy harvesters were also explored [123–125]. Such harvesters usually have an alterable resonant frequency because the equivalent stiffness or mass will change along with the changing of excitation conditions. Meanwhile, the working frequency range is tunable in a fixed range determined by the structure. In addition, the application environment is also limited by the structure of such harvesters (such as rotation environment, impacts, etc.).

One challenging issue is how to exactly control NEHs to vibrate at the highest orbit. The fact is that either mechanical or electrical method will inevitably consume external energy. However, how to get external energy in the real application environment is still a challenge, which should be much smaller than the harvested energy. Otherwise, it is not meaningful to use the active control method in vibration energy harvesting. This also raises the issue of the most interesting parameter to consider, that combine high probability of orbit jump and low power consumption. In that view, Huguet et al. showed the interest of using buckling level modification, as a low-cost yet efficient alternative [126].

Passively tunable MEHs have not been reported by far, while the adaptive structural design is required to have strong environmental robustness and high energy harvesting efficiency.

5.3. Uncertain factors

Optimization of vibration energy harvesters relies on the variation of geometrical parameters, electrical parameters of the circuit used in the energy harvesting process, and the external excitation. However, ambient vibrations in real applications often have time-varying characteristics (frequency, phase and amplitude), which can be considered as excitation errors or uncertainty of excitations. What's more, there may be errors and uncertainties in terms of processing, manufacturing, assembly, environmental influence, model hypotheses, etc., which lead to deviation of system parameters from the desirable values. The dynamic characteristics and output voltages of harvesters will be influenced and become uncertain [127]. For linear energy harvesters, Wang et al. [128] used a finite element level-based maximum entropy method to analyze the influence of random mass, stiffness, and electromechanical coupling coefficients on optimization design. For NEHs, Li et al. [129,130] used an improved interval extension based on the second-order Taylor's series to reveal the nature of influence of excitation conditions, whose accuracy was verified by the Monte Carlo simulation. They found nonlinear monostable energy harvesters are more sensitive to the excitation frequency than the excitation level. The robust design optimization method of nonlinear monostable energy harvesters with uncertain system parameters was presented. Huang et al. [131] used the Chebyshev polynomial approximation to analyze a stochastic NEH, and the non-negligible fluctuation of the output voltage was caused by random factor. Above mentioned methods are also suitable for bistable and multistable vibration energy harvesters to achieve optimal design and maximum power output.

By far, the effective optimization method for MEHs is still absent. In the future, we should firstly get the exact theoretical model of

NEHs, because the inaccuracy of the model will bring extra difficulty for optimization in the presence of uncertainties. In addition, exact analytical and numerical methods of the nonlinear model is of importance for predicting the output characteristics of the harvesters to realize effective optimization.

5.4. Size and output power

With the rapid development of manufacturing technology and material science, sensors and other embedded devices are becoming smaller and smaller. Their application environment also requires the harvester to have miniaturized dimensions and high energy density for powering the sensors. Microelectromechanical systems (MEMS) fabrication techniques were explored in miniaturization of vibration energy harvesters [132,133]. For example, Risquez et al. [134] developed nickel phosphorous (Ni-P) micromolding for manufacturing a 3D electrostatic energy harvesting MEMS. Shi et al. [135] optimized a structure of MEMS based polydimethysiloxane ferroelectret which can be used for harvesting energy from human body. Wang et al. [136] used the fan-folded structure to design a compact MEH ($5.28 \times 2.54 \times 1.73$ cm^3), and tested its energy harvesting performance under 1 g acceleration. The size is still too large in embedded application environment. For high-efficiency energy harvesting, the MEH is expected to realize large-amplitude interwell oscillations, which may also increase the physical space. This brings a challenge in miniaturization, while may also bring high energy output in compact size structure. In addition, advanced manufacturing processes, suitable materials and efficient optimization strategies are needful in miniaturization of MEHs.

For providing power source for high-power devices (such as high-power lighting installation, controllers, communication equipment, etc.), small-scale vibration energy harvesting (10 μW to 100 mW) is insufficient. Therefore, large-scale vibration energy harvesting (at least 1 W) from large-amplitude vibrations (such as the vibrations of high-rise buildings, automobile, boats, etc.) [137] has also received a lot of attention. For example, to harvest energy from human motion, Yuan et al. [138] designed a nonlinear mechanical motion rectification energy harvesting backpack (connected with an external resistor) which generated power larger than 3 W with a 13.6 kg load in experiments. Cassidy et al. [139] designed an electromagnetic energy harvesting used for energy harvesting from large buildings and bridges, and the power generated value can be larger than 5 W. Meanwhile, high power output inevitably requires the large size of harvesters. In addition, if MEHs have a higher energy harvesting efficiency and density than non-resonating energy harvesting systems subject to large-amplitude vibration is still unknown. At all events, designing a high-efficiency MEH based on specific application environment and target is necessary.

6. Discussion and perspectives

This paper reviews the working principle and progress, and presents perspectives of MEHs. In details, the governing model, approximate analytic methods are introduced, and different structural designs, experimental performance, influencing mechanism and optimization are illustrated as well as the applications of MEHs in rotational motion, wind energy harvesting, etc. Challenges about MEHs are also discussed. For the future research directions about MEHs, we present our ideas, as follows:

It is found that the MEH with more equilibrium positions can be designed to have shallower potential wells in the same physical space which are suitable for vibration energy harvesting from low-level excitations, and the effective frequency range is wider. Under high-level excitations, it is easy to realize interwell oscillations of MEHs. In this case, how to determine the number of equilibrium positions, and optimize the shape and depth of each potential energy well of MEHs is vital to increase the height of global interwell oscillation orbit thus leading to increasing the output voltage amplitude. In addition, environmental effects (temperature, humidness, etc.) will also bring challenges to the real applications of MEHs. Therefore, we need to design a MEH depending on the specific application conditions.

MEHs may have excellent performance subjected to complex multi-source excitations. For example, it may be interesting to explore the performance of MEHs installed on a vehicle wheel during actual driving in the full speed range of vehicles. The excitation from rotation motion can be considered as a harmonic excitation, while the excitation from the interaction between ground and the wheel may have random characteristics. The stochastic resonance of MEHs may be achieved, which leads to large output power.

When we design an interface circuit for MEHs, how to realize the maximum usable output power is a core problem. We should reveal the influence mechanism of interface or external circuits on the dynamic response and electromechanical coupling of MEHs. For example, if the interface circuit will lead to chaotic or intrawell oscillations of the harvester, and how to avoid this problem is crucial.

How to exactly control MEHs to vibrate at the highest orbit of importance for high-efficiency energy harvesting. Otherwise, their broadband advantage cannot be guaranteed. Using active control strategy requires that the consumed energy is not too high. Passively tunable MEHs that can be designed to have strong environmental robustness and high efficiency may partly increase the energy harvesting performance. In addition, the optimization design and the real application should fully consider uncertainties, as MEHs are highly sensitive to uncertainties.

For embedded application environment, advanced manufacturing processes, suitable materials and efficient optimization strategies are needful in miniaturization of MEHs. For large-scale vibration energy harvesting, how to design a MEH owning a high energy harvesting efficiency and density is of interest. At all events, designing a high-efficiency MEH based on the specific application environment and target is necessary.

MEHs, as a logical step towards high-efficiency, wide bandwidth energy harvester, do show a lot of attractive promises, but still require investigation on topics that can be inherited from original NEHs, or intrinsically new ones. Overall, MEHs have its unique characteristics, which are worthy of further investigating to promote the development and application of the vibration energy harvesting technique.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence this work reported in this paper.

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