Closure to "Discussion of 'On the Role of Nonlinearities in Energy Harvesting: A Critical Review and Discussion'" (Daqaq, M., Masana, R., Erturk, A., and Quinn, D. D., 2014, ASME Appl. Mech. Rev., 66(4), p. 040801)

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The authors would like to thank Professor Brian Mann for taking the time to read this paper and provide his insightful perspective into the role of nonlinearities in energy harvesting. Professor Mann has significant expertise in the field of energy harvesting and his commentary identifies several of the key advantages that result from the deliberate introduction of nonlinearities into energy harvesting devices. The goal of this closure is to complement his commentary by sharing additional thoughts that could be beneficial for the energy harvesting research community.

To begin, we would like to point out that the complexity of the response behavior of nonlinear harvesters as compared to their linear counterparts remains the biggest challenge preventing us from optimizing their performance and fully reaping their potential benefits. Nonlinear harvesters exhibit different behaviors that are not seen in linear systems including sub-harmonic, super-harmonic, quasi-periodic, aperiodic and chaotic responses. The long time response of the system depends on its initial conditions, and they can undergo different bifurcations in the parameter space as compared to those observed in linear systems, yielding sudden jumps in the response amplitude and/or switching in its period (doubling/halving). While we are currently able to show that, for some design parameters a nonlinear harvester can outperform a linear one, we are still unable to provide distinctive guidelines on how to properly design a nonlinear energy harvester for a given excitation source. Furthermore, we are still many steps away from designing electronic circuits specifically optimized to maximize the advantages of the nonlinearity and to properly condition the complex responses typical of their behavior.

Based on the research results reported in the open literature, we can say with confidence that the influence of nonlinearities on the performance of energy harvesters depends on the nature of the excitation source. If the excitation source is harmonic with a fixed frequency, the nonlinearity can be used to potentially decrease the sensitivity to uncertainties in the design parameters permitting the device to account for small variations in the excitation and/or natural frequency around their originally designed values. However, this advantage comes at an additional cost. Often, the nonlinearity yields coexisting steady-state responses with vastly different power outputs for a given excitation frequency. As a result, depending on the competing basins of attraction of these responses, the harvester can either provide high or low levels of output power. We agree with Professor Mann that this issue can be overcome by designing certain mechanisms that provide external input to guarantee that the harvester operates at its high power level capacity. However, as discussed in the manuscript, such mechanisms have yet to be thoroughly investigated and understood.

When the excitation source has Gaussian stationary random characteristics with a bandwidth much larger than that of the excitation (White Noise), a nonlinear harvester with a monostable potential energy function does not seem capable of offering any additional advantages over the linear design. However, when properly designed, based on the intensity of the input excitation, a harvester with a bistable potential well was shown to provide performance enhancements over the linear design. This, however, requires prior knowledge of the noise intensity because the optimal shape of the bistable potential is very sensitive to variations in the noise intensity. As a result, when the noise intensity changes, the mean output power drops significantly if the shape of the potential function is not adjusted accordingly. This, in the authors' opinion, constitutes a very interesting area for future research.

The nonlinearity seems to have its most benefits when the random excitation is colored, i.e., it has a bandwidth comparable to that of the harvester. Recently, Stanton et al. [1] illustrated these advantages by using Melnikov theory to find the combination of design parameters for which a bistable harvester can be designed to outperform the linear design. Masana and Daqaq [2] also illustrated experimentally that the bistable harvester is much less sensitive to changes in the center frequency, bandwidth, and intensity of the colored excitation than a monostable design.

To close, we note that few engineering examples exist where large nonlinearities are deliberately introduced to enhance performance [3]. The field of energy harvesting is certainly one such example, and has opened new avenues of research into the design of nonlinear systems. Performance benefits in harvesting devices can be achieved with the inclusion of phenomena that have been previously considered as undesirable or of lesser practical value. As such, we believe that nonlinear dynamics benefits from the problems arising in the field of energy harvesting as much as nonlinearities can be beneficial for the performance of energy harvesting systems.

References

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Manuscript received February 19, 2014; final manuscript received February 19, 2014; published online May 2, 2014. Editor: Harry Dankowicz.

