On the energy harvesting potential of piezoeaeroelastic systems

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This paper investigates the concept of piezoeaeroelasticity for energy harvesting. The focus is placed on mathematical modeling and experimental validations of the problem of generating electricity at the flutter boundary of a piezoeaeroelastic airfoil. An electrical power output of 10.7 mW is delivered to a 100 kΩ load at the linear flutter speed of 9.30 m/s (which is 5.1% larger than the short-circuit flutter speed). The effect of piezoelectric power generation on the linear flutter speed is also discussed and a useful consequence of having nonlinearities in the system is addressed. © 2010 American Institute of Physics. [doi:10.1063/1.3427405]

Vibration-based energy harvesting using piezoelectric transduction has been investigated by several researchers over the past decade.1 Typically, cantilevers with piezoceramics are used as piezoelectric energy harvesters and the source of excitation is assumed to be base motion.2–6 Researchers have investigated linear2,3 and nonlinear4–6 piezoelectric energy harvesting using cantilevered plates under airflow excitation.10 Therefore, the model and the experiments are their physical simplicity and the fundamental insight they provide.11

Piezoelectric energy harvesting from aeroelastic vibrations has been studied by a few authors and limited archived work exists. The literature includes investigations of energy harvestings has been studied by a few authors and limited archived work exists. The literature includes investigations of energy harvestings.12–14 Later at other conferences, Bryant and Garcia discussed energy harvesting using piezoelectric cantilevered plates.15–18

This paper presents an experimentally validated piezoeaeroelastic model with a focus on the generated electrical power and its effect on the aeroelastic response. Lumped-parameter wing-section models are very appealing due to their physical simplicity and the fundamental insight they provide.19 Therefore, the model and the experiments are given for a modified typical section undergoing self-sustained oscillations at the neutral stability condition. While the analysis given here is linear, a useful consequence of having nonlinearities in the piezoeaeroelastic system is also discussed.

Consider the piezoeaeroelastic section under airflow excitation shown in Fig. 1. After introducing piezoelectric coupling to the plunge degree of freedom (DOF) in addition to two structural damping coefficients and considering a resistive load in the electrical domain, the lumped-parameter aeroelastic equations12 are modified to obtain the following piezoeaeroelastic equations:

\[ m \ddot{x} + b \ddot{\alpha} + d_x \dot{\alpha} + k_x \alpha + k_a \alpha = M, \]  

\[ C_p \ddot{v} + v/R_l + \theta \dot{\theta} = 0, \]  

where \( h \) is the plunge displacement (translation), \( \alpha \) is the pitch displacement (rotation), \( m \) is the airfoil mass per length (in the span direction), \( m_j \) accounts for the fixture mass per length in the experiments connecting the airfoil to the plunge springs \( (m_j = 0) \) for the ideal representation given in Fig. 1, \( I_p \) is the moment of inertia per length about the reference point \( P \) where \( h \) is measured, \( b \) is the semichord length, \( \ell \) is the span length (into the page), \( x_a \) is the dimensionless chordwise offset of the reference point from the centroid (point \( C \)), \( k_h \) is the stiffness per length in the plunge DOF, \( k_a \) is the stiffness per length in the pitch DOF, \( L \) is the aerodynamic lift per length, \( M \) is the aerodynamic pitching moment per length, \( d_h \) and \( d_a \) respectively, are the structural damping coefficients in the plunge DOF and the pitch DOF, \( R_l \) is the load resistance, \( v \) is the voltage across the resistive load, \( C_p \) is the equivalent capacitance of the piezoceramic layers, and \( \theta \) is the electromechanical coupling term and an over-dot represents differentiation with respect to time.

Assuming harmonic response at frequency \( \omega \) (i.e., \( h = h e^{j \omega t}, \alpha = \bar{\alpha} e^{j \omega t}, v = v e^{j \omega t}, L = \bar{L} e^{j \omega t}, \) and \( M = M e^{j \omega t} \)) where \( j = \sqrt{-1} \) leads to the following complex eigenvalue problem for the steady-state plunge and pitch displacements:

\[ \beta \bar{h} + \frac{\ell_h}{\mu} - \kappa(\omega) - \alpha^2(1 + j \gamma_h) \lambda \bar{h} + \left( x_a + \frac{\ell_a}{\mu} \right) \bar{\alpha} = 0, \]  

FIG. 1. (Color online) Schematic of a piezoeaeroelastic section under uniform airflow.

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where the aerodynamic loads (\( \ell_p, \ell_a, m_p, m_a \)) are taken from Theodorsen’s unsteady thin airfoil theory\(^1\) and are functions of the reduced frequency \( k = \omega b / U \) (where \( U \) is the airflow speed) and the geometric parameters. It is important to note that, in this linear model, the harmonic response assumption holds for the condition of neutral stability only \( \text{i.e., Eqs. (4) and (5) are valid at the classical flutter boundary only} \). The dimensionless terms are the complex eigenvalue, \( \lambda = (\omega_a / \omega)^2 \), the frequency ratio, \( \sigma = \omega_h / \omega_a \) (where \( \omega_h = \sqrt{k_b / m} \) and \( \omega_a = \sqrt{k_a / I_p} \)), the dimensionless radius of gyration, \( r = \sqrt{l_p / \rho b^2} \), the airfoil-to-affected mass ratio, \( \mu = m / \rho b^2 \) (where \( \rho \) is the free-stream air mass density), and a mass ratio that accounts for the presence of a fixture between the airfoil and the plunge springs, \( \beta = (m + m_p) / m \). The loss factors in Eqs. (4) and (5) are assumed to obey \( \gamma_h = \omega d_l / k_b \) and \( \gamma_a = \omega d_a / k_a \), and they are identified at zero airflow speed.

The dimensionless term \( \kappa(\omega) \) in Eq. (4) is due to eliminating the voltage term using Eq. (3) in Eq. (1) and it depends on the eigenvalue \( \lambda \) since it is a function of frequency: \( \kappa(\omega) = j \theta (j \omega C_{\text{pf}} + 1 / R_c)^{-1} / (\omega m \ell) \). Hence an iterative solution procedure is required where the frequency to be used in \( \kappa(\omega) \) is obtained from the eigenvalue that becomes unstable with increasing airflow speed. The convergence of the iterative eigensolution is extremely fast if one starts with the solution of the piezoelectrically uncoupled aeroelastic problem \( [\kappa(\omega) = 0] \). Once the complex eigenvector relationship between \( \vec{h} \) and \( \vec{\alpha} \) is obtained, \( \vec{v} \) is calculated using

\[
\vec{v} = -j \omega \theta (j \omega C_{\text{pf}} + 1 / R_c)^{-1} \vec{h}.
\]

For a given load resistance, the airflow speed that makes the imaginary part of the respective eigenvalue branch zero is the linear flutter speed \( (U = U_f) \) and the piezoelectrically eigenvector \( \{ \vec{h} \; \vec{\alpha} \; \vec{v} \} \) is obtained using this eigenvalue at this particular speed.

Figure 2 shows the experimental setup used for investigating the piezoelectric response of an airfoil section. The system parameters are \( x_p = 0.260, r = 0.504, \beta = 2.597, \sigma = 3.33, \mu = 29.6, b = 0.125 \text{ m}, \ell = 0.5 \text{ m}, \) and \( \omega_n = 15.4 \text{ rad/s} \). The loss factors identified for the plunge DOF and the pitch DOF at zero airflow speed are \( \gamma_p = 0.007 \) and \( \gamma_a = 0.12 \). A dimensionless geometric parameter required for the Theodorsen function\(^1,1\) \( \ell_p / b \) is the relative location of the reference point with respect to the midchord and it is \( a = -0.5 \) for this setup. The plunge stiffness of the airfoil is due to four steel beams (in clamped-clamped end conditions) connecting the airfoil to the ground from the reference point. Two PZT-5A piezoceramics (QP10N from Midé Technology Corporation) are attached onto the roots of two of these beams symmetrically and their electrodes are combined in parallel. The electromechanical coupling term is obtained based on distributed-parameter modeling as \( \theta = 1.55 \text{ mN/V} \) and the manufacturer’s published equivalent capacitance of \( C_{\text{pf}} = 120 \text{ nF} \) is used in the model. In the experiments, the airflow speed is slowly increased from zero until almost persistent piezoelectric response is obtained for each resistive load.

The short-circuit \( (R_l \rightarrow 0) \) and the open-circuit \( (R_l \rightarrow \infty) \) flutter speeds are measured as \( U_{c} = 8.85 \text{ m/s} \) and \( U_{c} = 8.90 \text{ m/s} \), respectively. Figure 3 shows the piezoelectric response for an electrical load resistance of 100 k\( \Omega \) with almost persistent oscillations approximately at the linear flutter speed of 9.30 m/s. Among the set of resistors used in the experiments, this is the electrical load that gives the maximum power output \( (10.7 \text{ mW}) \). For this electrical load, the absolute value of the normalized piezoelectric eigenvalue is obtained from the model as \( |\vec{h} / |\vec{v}| / |\vec{\alpha}| = 4.18 / 7.65 \) at the flutter speed of 9.56 m/s. The experimental maximum response amplitudes in Fig. 3 are \( |\vec{h}| = 7.65 \text{ mm}, |\vec{\alpha}| = 4.18^\circ, \) and \( |\vec{v}| = 32.7 \text{ V} \). Hence the experimental ratios \( |\vec{\alpha}| / |\vec{h}| = 4.18 / 7.65 = 0.55^\circ / \text{mm} \) and \( |\vec{\alpha}| / |\vec{h}| = 32.7 / 7.65 = 4.27 \text{ V/mm} \) exhibit good agreement with the model.

The linear flutter speed for \( R_l = 100 \text{ k}\Omega \) (close to short-circuit conditions) is predicted by the model as 9.06 m/s, overestimating the experimental value of 8.85 m/s by 2.4%. The model predicts the linear flutter speed for \( R_l = 100 \text{ k}\Omega \) as 9.56 m/s, which overestimates the experimental value of 9.30 m/s by 2.8%. The flutter frequency is obtained by the model as 5.17 Hz for \( R_l = 100 \text{ k}\Omega \) (underestimating the experimental value by 1.7%) and as 5.14 Hz for \( R_l = 100 \text{ k}\Omega \) (underestimating the experimental value by 2.3%).

Figure 4(a) shows the voltage-to-plunge displacement amplitude ratio while Fig. 4(b) shows the pitch-to-plunge displacement amplitude ratio for the set of resistors used in the experiment along with the theoretical predictions. The voltage-to-plunge displacement versus load resistance curve exhibits linear asymptotes similar to the trend in the harmonic base excitation of piezoelectric energy harvesters\(^2\).

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**Fig. 2.** (Color online) Experimental setup showing a typical aeroelastic section with piezoceramics attached onto the plunge stiffness members.

**Fig. 3.** (Color online) Experimental piezoelectric response for \( R_l = 100 \text{ k}\Omega \) and \( U_c = 9.30 \text{ m/s} \).
Theoretical and experimental data points are given for the flutter speed that

due to the shunt damping effect\(^\text{15}\) of piezoelectric power generation. The ex-

terior, piezoelectric energy harvesting has the favorable effect

\[ v_{\infty} = 2.42 \text{ V} \]

whereas for \( R_l = 100 \text{ k}\Omega \), \( |\bar{h}| = 5.15 \text{ mm}, |\bar{a}| = 2.82^\circ \), and \( |\bar{v}| = 4.40^\circ \). There-

\[ \bar{v} = 83.1 \text{ V} \]

\[ \bar{v} = 4.40^\circ \]

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\[ U_{\infty} = 7493 \text{ mph}, 2009 \]